

Exercise 27

Explain, using Theorems 4, 5, 7, and 9, why the function is continuous at every number in its domain. State the domain.

$$Q(x) = \frac{\sqrt[3]{x-2}}{x^3-2}$$

Solution

The numerator is $\sqrt[3]{x-2}$, a root function, which is continuous according to Theorem 7. The denominator is $x^3 - 2$, a polynomial, which is continuous according to Theorem 7. $Q(x)$ is the ratio of these functions, which according to Theorem 4 is also continuous wherever the denominator is not zero.

$$x^3 - 2 \neq 0$$

$$x^3 \neq 2$$

$$\sqrt[3]{x^3} \neq \sqrt[3]{2}$$

$$x \neq \sqrt[3]{2}$$

Therefore, the domain is

$$\left(-\infty, \sqrt[3]{2}\right) \cup \left(\sqrt[3]{2}, \infty\right).$$